Reg. No.: | | | | | | | |

G. VENKATASWAMY NAIDU COLLEGE (AUTONOMOUS), KOVILPATTI – 628 502.

PG DEGREE END SEMESTER EXAMINATIONS - NOVEMBER 2024.

(For those admitted in June 2023 and later)

PROGRAMME AND BRANCH: M.Sc., MATHEMATICS

SEM	CATEGORY	COMPONENT	COURSE CODE	COURSE TITLE
I	PART - III	CORE-1	P23MA101	ALGEBRAIC STRUCTURES

Date : 04.11.2024/AN **Time : 3 hours** **Maximum: 75 Marks**

Course Outcome	Bloom's K-level	Q. No.	**SECTION – A (10 X 1 = 10 Marks)** **Answer ALL Questions.**
CO1	K1	1.	If $O(G)$ = _____ , then G is abelian. a) p^3 b) p^2 c) p d) p^4
CO1	K2	2.	The number of conjugate classes in S_n is _________. a) $p(n+1)$ b) $p(n^2)$ c) $p(n)$ d) $p(n-1)$
CO2	K1	3.	If G is solvable group and if $\bar{G}$ is a ________ of G, then $\bar{G}$ is solvable. a) isomorphic image b) subgroup c) subset d) homomorphic image
CO2	K2	4.	An $R-module$ M is said to be cyclic if every element $m \in M$ is of the form ________. a) $m = rm_0$ b) $m = -rm_0$ c) $m = 2rm_0$ d) $m = r^2m_0$
CO3	K1	5.	The subspace W of V is invariant under $T \in A(V)$ if _______. a) $W \subset TW$ b) $WT \subset W$ c) $T \subset W$ d) $W \subset T$
CO3	K2	6.	The linear transformation $S, T \in A(V)$ are said to be similar if there exists an invertible element $C \in A(V)$ such that __________. a) $S = CTC^{-1}$ b) $S = TC^{-1}$ c) $T = CSC^{-1}$ d) $T = SC^{-1}$
CO4	K1	7.	If $E^2 = E$ and $F^2 = F$ and they are similar iff they have the _______. a) different nullity b) different rank c) same nullity d) same rank
CO4	K2	8.	A matrix is said to be diagonalizable if it is similar to a ______ matrix. a) diagonal b) orthogonal c) unitary d) symmetric
CO5	K1	9.	A matrix A is said to be a skew symmetric if _______. a) $A' = A^T$ b) $A' = A$ c) $A' = -A^T$ d) $A' = -A$
CO5	K2	10.	If A is invertible then $tr(ACA^{-1})$ = ___________. a) trC b) trA c) trC^{-1} d) trA^{-1}
Course Outcome	**Bloom's K-level**	**Q. No.**	**SECTION – B (5 X 5 = 25 Marks)** **Answer ALL Questions choosing either (a) or (b)**
CO1	K2	11a.	Prove that $N(a)$ is a subgroup of G. **(OR)**
CO1	K2	11b.	Prove that $n(k) = 1 + p + \cdots + p^{k-1}$.
CO2	K2	12a.	Prove that S_n is not solvable for $n \geq 5$. **(OR)**
CO2	K2	12b.	Suppose that G is the internal direct product of $N_1, \dots N_n$. Then prove that for

			$i \neq j, N_i \cap N_j = (e)$ and if $a \in N_i, b \in N_j$ then also prove that $ab = ba$.
CO3	K3	13a.	Explain the proof of "If V is n −dimensional over F and if $T \in A(V)$ has all its characteristic roots in F, then T satisfies a polynomial of degree n over F". **(OR)**
CO3	K3	13b.	Explain the proof of "If $u \in V$, is such that $uT^{n_1-k} = 0, where\ 0 < k \leq n_1$, then $u = u_0T^k$ for some $u_0 \in V_1$".
CO4	K3	14a.	Write the proof of "Suppose the two matrices A, B in F_n are similar in K_n where K is an extension of F .Then A and B are already similar in F_n". **(OR)**
CO4	K3	14b.	Write the proof of "Suppose that $V = V_1 \oplus V_2$, where $V_1, V_2$are subspace of V invariant under T.Let T_1 and T_2 be linear transformations induced by T on V_1 and V_2 respectively. Then the minimal polynomial for T over F is the least common multiple of $p_1(x)$ and $p_2(x)$ where $p_1(x)$ and $p_2(x)$ is the minimal polynomial of T_1 and T_2 respectively".
CO5	K4	15a.	Analyze the proof of "If $T \in A(V)$ then prove that trT is the sum of the characteristic roots of T". **(OR)**
CO5	K4	15b.	Analyze the proof of "If $(vT, vT) = (v, v)$ for all $v \in V$, then prove that T is unitary".

Course Outcome	Bloom's K-level	Q. No	**<u>SECTION – C</u> (5 X 8 = 40 Marks)** **Answer <u>ALL</u> Questions choosing either (a) or (b)**
CO1	K4	16a.	Illustrate Cauchy's theorem. **(OR)**
CO1	K4	16b.	Illustrate Sylow's theorem.
CO2	K5	17a.	Prove: If $p(x) \in F[x]$ is solvable by radicals over F, then the Galois group over F of $p(x)$ is a solvable group. **(OR)**
CO2	K5	17b.	Prove: Every finite abelian group is the direct product of cyclic groups.
CO3	K5	18a.	Explain the proof of "If $T \in A(V)$ has all its characteristic roots in F, then there is a basis of V in which the matrix of T is triangular". **(OR)**
CO3	K5	18b.	Explain the proof of "Two nilpotent linear transformations are similar iff they have the same invariants".
CO4	K5	19a.	Explain the proof of "The elements S and T in $A_F(V)$ are similar in $A_F(V)$ iff theyhave the same elementary divisors". **(OR)**
CO4	K5	19b.	Explain the proof of "For each i=1,2,…,k, $V_i \neq (0)$ and $V = V_1 \oplus V_2 \oplus \dots \oplus V_k$. Then the minimal polynomial of T_i is $q_i(x)^{l_t}$".
CO5	K6	20a.	Discuss the proof of "The linear transformation T on V is unitary iff it takes an orthonormal basis of V into an orthonormal basis of V." **(OR)**
CO5	K6	20b.	Discuss the proof of For all $A, B \in F_n$, i) $(A')' = A$ ii) $(A + B)' = A' + B'$ iii) $(AB)' = B'A'$